

Comments on “Lattice Formulation of the Standard Model” by Creutz. et. al.

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Abstract

The main construction of the paper in the title is summarized using more standard particle physics language. Also, a flaw is pointed out and several objections to the more general thoughts expressed by Creutz et. al. are raised.

Creutz et. al. [1] consider a model in which every fermion of the standard model is mirrored by an additional fermion differing only in handedness. One extra right handed neutrino is added per generation. The discussion is mainly restricted to a single generation. Clearly, the set-up calls for $SO(10)$ GUT notation [2]. More specifically, the chain $SO(10) \rightarrow SU(4)_{PS} (\approx SO(6)) \otimes [SU(2)_L \otimes SU(2)_R] (\approx SO(4))$ is relevant, with PS referring to Pati-Salam. The one generation 16 decomposes into $(4, 2, 1) \oplus (\bar{4}, 1, 2) \equiv \Psi^L \oplus \Psi^R$. The central object in [1] is a dimension six Baryon number violating operator. For a single generation there are four possibilities [3]. Creutz et. al. make what amounts to a choice of $\mathcal{O}^{(3)}$ in Weinberg's notation [3]. Using the notation of [1] for the fermions (except for the explicit handedness), introduce the scalar fields $\Phi_{\alpha\beta}^L \equiv \Psi_{\alpha is}^L \Psi_{\beta jt}^L \epsilon_{ij} \epsilon_{st} = -\Phi_{\beta\alpha}^L$ and similarly for L replaced by R . $\Phi^{L,R}$ are singlets under both the left and right $SU(2)$'s. For $(\alpha\beta)$ restricted to $SU(3)$ -color $\Phi_{\alpha\beta}^L$ is a $\bar{3}$ and for $\beta = 4$ it is a 3. Under $SU(4)_{PS} (\approx SO(6))$ Φ^L is an antisymmetric rank two tensor (six component vector). Φ^L can be decomposed into selfdual and anti-selfdual tensors $\hat{\Phi}_{\alpha\beta}^{L\pm} = \Phi_{\alpha\beta}^L \pm \frac{1}{2}\epsilon_{\alpha\beta\gamma\delta}\Phi_{\gamma\delta}^L$. The Baryon number violating operator is a scalar under $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ given by $V_L = \epsilon_{\alpha\beta\gamma\delta}\Phi_{\alpha\beta}^L\Phi_{\gamma\delta}^L$. In terms of $\hat{\Phi}^{L\pm}$, V_L is diagonal. Similar definitions give V_R . The main idea is to add to the Lagrangian a term $g[V_L + V_R + h.c.]$ for each generation and only for the mirrors. V_L could be replaced by coupling an auxiliary field $\phi_{\alpha\beta}^L$ to $\Phi_{\alpha\beta}^L$, making the Lagrangian bilinear in the Ψ 's.

First, in the spirit of the "Yukawa approach" to the regularization of chiral gauge theories ([1]), the theory is studied in the absence of gauge fields. On the lattice, if g is strong enough, and one ignores coupling between the mirrors and the ordinary fermions, only the mirrors become massive and their masses are large. It is hoped that this will persist to weaker g 's and that the theory stay in a symmetric phase ($\langle \Phi_{\alpha\beta}^{L,R} \rangle = 0$). Then, one hopes that when the ordinary $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ gauge interactions are turned on, the ordinary Higgs mechanism would be *necessary* to render massive the W 's, the Z and the fermions.

The mirrors and the ordinary fermions interact via $L - R$ couplings through a chain of heavy Dirac fermions, along a fictitious fifth dimension as suggested by Kaplan [4]. On the lattice this is a finite segment and it is hoped that it will prevent the mirror masses from feeding down to the ordinary fermions. Some of the anomalous global symmetries carried by ordinary particles in the continuum are not exact on the lattice since the mirrors do not have strictly infinite masses and the presence of $V_L + V_R$ is felt. This method of eliminating unwanted symmetries is due to Eichten and Preskill [5]. It is hoped that in the presence of an $SU(2)_L$ instanton the correct 't Hooft vertex [6] will appear in the continuum limit independently of the strength of g .

Actually, the model of [1] is invariant under a discrete $Z_3 \otimes Z_3$ symmetry: $u_R, d_R \rightarrow z_R u_R, z_R d_R$ and $u_L, d_L \rightarrow z_L u_L, z_L d_L$ ($z_{R,L}^3 = 1$), independently for each generation. The true standard model would not obey this symmetry due to color-instantons. Full compliance with the Eichten-Preskill guidelines would forbid this discrete global symmetry.

Although the construction of [1] is specific to the standard model the authors make some general remarks. As a chiral gauge theory the standard model is relatively simple, as the single “dangerous” group is a $U(1)$. Moreover, the related coupling isn’t asymptotically free. It is unclear then how much can be inferred about the general problem of regulating an asymptotically free non-abelian chiral gauge theory in four dimensions. The authors view as the main weakness of their approach the questionable existence of the appropriate phase in the ungauged model. “Such a situation would cast serious doubts on any construction of chiral gauge theories” they say referring to the possibility of the desired phase being “squeezed out”. In view of the many arbitrary choices they made (one particular Baryon number violating operator, a left-right symmetric model prior to gauging, a mirror-Yukawa approach [7], and the generic failure rate in simpler Yukawa models) it is difficult to justify the “any” in their statement.

The most confusing statements in [1] are made comparing their proposal to the “overlap” [8]. The latter is a general scheme which can be interpreted as treating an infinite number of lattice fermions. However, there is nothing infinite in the overlap regularization. Nevertheless, Creutz et. al. conclude that their approach is “cleaner in that gauge invariance is exact, all infinities are eliminated, and the requirement of anomaly cancelation is manifest”.

The manifest requirement for anomaly cancelation is justified elsewhere in [1]: it simply means that the charged Lepton charge has to be 3 times the quark charge to ensure gauge invariance of the vertex. Anomaly cancelation is not *directly* required, but just happens to hold because, for example, we don’t have more charged fermions that don’t participate in the vertex. Working in an $SO(10)$ scheme guaranteed anomaly cancelation from the start. As a matter of fact, a ’t Hooft vertex is explicitly used in the overlap in order to completely define the model (section 5.2 of [8]). Thus, the overlap is similar to [1] in this respect. But, [1] has built in $\Delta B = \Delta L = 1$ for Baryon and Lepton number violations. Instantons only allow $\Delta B = \Delta L = 3$ and the overlap has clear preference for these processes. Also, the overlap would produce no unwanted global symmetries, global or discrete.

If one applied the overlap to the standard model one would have to integrate over a degree of freedom $e^{i\theta}$ for each site on the four dimensional lattice. The θ ’s can be interpreted as a gauge degree of freedom - somewhat similar to a longitudinal photon. It is

hoped [8] that the integration over θ would restore $U(1)$ invariance without adding extra nonlocal terms, similarly to [9]. If the $U(1)$ were anomalous this cannot happen [10]. So, anomaly cancelation plays a much more intrinsic role in the overlap than in [1]. In [1], similarly to the Yukawa approach, the phase structure of the ungauged theory is deemed crucial. This phase structure cannot be influenced by which group we intend to gauge, so the phases are hardly dependent on anomaly cancelation. It is *a priori* possible to find the “right” phase, while some particular gauging still shouldn’t work because anomalies do not cancel. In practice, to implement the proposal of [1], an integration over some auxiliary fields, like $\phi_{\alpha\beta}^{L,R}$, will always be needed. In the overlap, the θ ’s at least do not carry color or charge.

The overlap definition includes the above gauge averaging, so there is an almost tautological gauge invariance. Thus, exact gauge invariance isn’t really an issue. What does matter though are the “hopes” about the degrees of freedom surviving the continuum limit: In the overlap one hopes that with anomaly cancelation the variables θ will stay massive and decouple as in [9]. Creutz et. al. hope that the whole slew of mirrors, (possibly bound into Φ ’s ?) will decouple. If, for example, Φ acquires an expectation value Creutz et. al.’s model would loose ordinary confinement. It should sound strange that the strong interactions are put in danger as a result of trying to gauge the weak $U(1)$. In the overlap, $SU(3)$ -color would be well isolated.

The overlap has been subjected to a real, albeit modest, test [11]: In an exactly soluble abelian two dimensional chiral model the ’t Hooft vertex was shown to come out correctly. Thus, the hopes about the overlap have been shown to come true at least in one non-trivial instance. Nothing has been reported that compares even remotely for any example of the Yukawa approach, [1] included. Creutz et. al. describe the model in [11] as employing a “tricky twist” because the ’t Hooft vertex contains a derivative. In any dimension, the vertex contains one fermion field for each fermionic zero mode, all at the same point in Euclidean space. Quite often, derivatives will be needed to make the vertex a Lorentz scalar. In four dimensions this happens, for example, in $N=1$ supersymmetric pure $SU(2)$ gauge theory [12]. A proposal that cannot work when there are derivatives in the ’t Hooft vertex is unreasonably restricted. While two dimensions differ substantially from four, numerical, dynamical, fully non-perturbative tests of any proposal are practical only in two dimensions (excluding odd dimensions) at present. The issues related to chirality specifically are reasonably similar in all even dimensions and it is difficult to accept that a proposal could work in four dimensions but fail in two.

In the closing paragraph of [1] it is suggested that a success of their proposal would justify several other approaches, among them the overlap. Indeed, the fifth dimension, be-

ing of arbitrary length, could be taken to infinity yielding something similar to the overlap, now with trivial gauge averaging. But, the overlap, when interpreted as a theory containing an infinite number of fermions also contains an infinite subtraction of the effective action due to the infinitely many heavy particles. Including such a subtraction will always mar a “pure” action interpretation where ghost fields are excluded. A success of the proposal of [1] cannot justify some of the other proposals they mention: unlike the overlap, most of them do contain explicit infinities and ignore instantons.

In conclusion, the approach proposed in [1] is too special to allow drawing any general lessons about defining non-perturbatively chiral gauge theories, or about other approaches to the problem. In its present form the construction of [1] has a potential flaw, an unwanted global discrete symmetry. In practice, it is unlikely we’ll know in the foreseeable future whether the full four dimensional proposal of [1] works or not. Even if the desired phase were to be ruled out in some simpler variant, we couldn’t attach a general meaning to the finding. Since the authors discount two dimensional tests, the prospects for any objective evidence of success in their approach are slim.

Acknowledgments.

This research was supported in part by the DOE under grant # DE-FG05-96ER40559.

References.

- [1] M. Creutz et. al., hep-lat/9612017.
- [2] H. Georgi, D.V. Nanopoulos, Nucl. Phys. B155, 52 (1979).
- [3] S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979); F. Wilczek, A. Zee, Phys. Rev. Lett. 43, 1571 (1979); L. F. Abbott, M. B. Wise, Phys. Rev. D22, 2208 (1980).
- [4] D. Kaplan, Phys. Lett. B288, 342 (1992).
- [5] E. Eichten, J. Preskill, Nucl. Phys. B268, 179 (1986).
- [6] G. ’t Hooft, Phys. Rev. Lett. 37, 8 (1976).
- [7] I. Montvay, Phys. Lett. 199B, 89 (1987).
- [8] R. Narayanan, H. Neuberger, Nucl. Phys. B443, 305 (1995).
- [9] D. Foerster, H. B. Nielsen, M. Ninomiya, Phys. Lett. 94B, 135 (1980).
- [10] J. Preskill, Ann. Phys. 210, 323 (1991).
- [11] R. Narayanan, H. Neuberger, hep-lat/9609031, Phys. Lett. B, to appear;
Y. Kikukawa, R. Narayanan, H. Neuberger, hep-lat/9701007, Phys. Lett. B, to appear; hep-lat/9705006.
- [12] A. I. Vainshtein, V. I. Zakharov, JETP Lett. 35, 323 (1982).